**Notes: Week of Sept28.Fall2012**

Course website: [www.cis.syr.edu/~sueo/cis275](http://www.cis.syr.edu/~sueo/cis275)

More practice with proofs

**Claim:**  Let A, B, C be sets. If A ⊈ B and C is not the empty set, then AxC ⊈ BxC

**Proof:** Suppose A ⊈B and C is not the empty set

**[NTS:** AxC ⊈ BxC **]**

Because A ⊈ B, there exists some x ∈ A such that x ∉ B. Because C is not the empty set, there exists some y ∈ C. Thus (x,y) ∈ AxC but (x,y) ∉ BxC. Thus AxC ⊈ BxC and hence claim is true.

**Claim:** Let A,B be sets. Then P(A\B)\{ ∅ } ⊆ P(A) \ P(B)

**Proof**: Consider arbitrary X ∈ P(A\B)\{ ∅ }

**[NTS:**  X ∈ P(A) \ P(B) **]**

Thus X ∈ P(A\B) and X ∉ { ∅ }

Hence we know:

X ⊆ A\B

X is not ∅, *therefore it contains some element*

Consider arbitrary w ∈ X

Because X ⊆ A\B, w ∈ A and w ∉ B.

Because w is arbitrary X ⊆ A and thus X ∈ P(A)

Because w ∉ B, X ⊈ B and thus X ∉ P(B)

Therefore X ∈ P(A)\ P(B)

Because X was arbitrary, P(A\B)\{ ∅ } ⊆ P(A) \ P(B)

Collection of hypotheses:

H1, H2, …., Hk *(where k>=0)*

Desired conclusion/consequence:

C

( H1 ^ H2 ^ … ^ Hk )→C

Direct proof: suppose hypotheses true, show conclusion true

Proof by contraposition:

Assume ¬C

Show ¬( H1 ^ H2 ^ … ^ Hk )

**Claim**: Let n be an integer. If n2 is even, then n is even.

**Proof Attempt 1(direct)**

Suppose n2 is even

[**NTS:** n is even]

If n2 is even, there exists some integer k such that n2 = 2\*k

n is the square root of 2k

**Proof Attempt 2(contraposition)**

Assume n is not even( n is odd)

[NTS: that it is not the case that n2 is even]

Because n is odd, there exists some integer k such that n = 2k +1

By algebra

n2 = (2k+1)2

= 4k2+4k+1 = 2(2k2+2k) +1

Because k is an integer, 2k2+2k is an integer, and thus n2 is odd

Thus the original claim is true.

**Claim:**  Let A,B be sets. If A x B is infinite then either A is infinite or B is infinite

**Proof (by contraposition)**

Suppose both A and B are finite

Because both sets are finite, there exist natural numbers m,n such that |A|=m and |B|=n

Then |AxB| = m\*n, which is a natural number, therefore AxB is finite.

By contraposition, the original claim is true

**Proof by contradiction**:

Assume the hypotheses are true and the conclusion is false

Show that a contradiction results

**Claim:** Let n b an integer. If n is even, then n is not odd

**Proof (by contradiction)**:

Suppose n is even and it is not the case that n is not odd (ie, n is odd)

[NTS: contradiction]

Because n is even, there exists an integer k such that n=2k,

Because n is odd there exists an integer j such that n = 2j+1

Thus by algebra,

2k=2j+1

And hence k=(2j+1)/2 = j + (1/2)

Which means j = k – (1/2)

Thus k,l cannot both be integers which contradicts earlier assertion

Thus by contradiction, the original claim is true.

*Definitions:*

*A number is rational iff it can be expressed in the for p/q where both are integers and q!=0*

**Claim** sqrt(2) is irrational:

**Proof** (by contradiction)

Suppose sqrt(2) is rational

[NTS: contradiction]

By def’n of rational, there exist integers p,q (q is not 0), such that sqrt(2)=p/qand furthermore p and q can be chosen so that their only common divisor is 1

By algebra, 2= p2/q2, which means p2=2q2 therefore p2 is even

Hence p is even and there exists an integer k such that p=2k

Therefore q2 = (2k)2 / 2 = 4k2/2= 2k2

Thus q is even , since both p and q are even they share the divisor 2, contradicting that their common devisor is 1.

By contradiction, the claim is correct